

Performance Analysis of QPSK in Co-Channel Interference and Fading

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ABSTRACT: The performances of QPSK in the presence of co-channel interference in both nonfading and fading environments are analyzed. Three approaches for representing the co-channel interference are investigated. These are a precise error probability method, a sum of sinusoids (sinusoidal) model, and a Gaussian interference model. In addition to determining precise results for the performance of QPSK in co-channel interference, it is our aim in this paper to examine the validity of these two interference models in both additive white Gaussian noise (AWGN) environments and in different flat fading environments; Rayleigh, Ricean, and Nakagami. In some previous work the effects of inter symbol interference (ISI) produced by the co-channel interference were neglected, as were also the symbol timing offsets of the different signals. In this paper, Nyquist pulse shaping is considered and the effects of cross channel ISI produced by the co-channel interference are accounted for in the precise interference model. Two performance criteria are considered. These are the average bit error rate and the interference penalty. The latter is defined as the increase in signal-to-noise power ratio (SNR) required by a system with co-channel interference in order to maintain the same BER as a system without interference. In this environment, the fading experienced by the interfering signals may be represented by a Rayleigh-fading model while the fading experienced by the desired signal may be represented by a Ricean or a Nakagami-fading model.

Keywords: Noise Models, System Models, Error Rates

1. INTRODUCTION

In recent years, mobile communications has become very popular and the demand for its services has increased significantly. The capacity of mobile communication systems is mainly limited by co-channel interference caused by frequency reuse. The acceptable co-channel interference at the receiver determines the minimum allowable distance between adjacent co-channel users and hence the system capacity. So, in order to increase the capacity we need to model co-channel interference. Transmission in wireless communication systems is carried out in a radio propagation environment. The radio propagation environment places fundamental limitations on the performance of wireless communications systems. Transmission is subjected to two major sources of degradation. These are fading and co-channel interference. Fading is due to multipath propagation and co-channel interference is due to the reuse of the

radio frequencies. The aim of the frequency reuse is to increase the spectrum efficiency. Therefore, analyzing the performances of digital communications on fading channels in the presence of co-channel interference is of considerable interest.

Recently, the use of linear modulation schemes such as quaternary shift keying (QPSK) in wireless communication systems has received much attention. This is mainly because linear modulation schemes are more spectrally efficient than constant envelope modulation schemes such as Gaussian minimum shift keying modulation (GMSK). The increasing demand for spectrum resources has motivated interest in using coherent demodulation. It was shown in [1] that improvements as much as 30% in spectrum efficiency might be obtained by using coherent rather than differential demodulation. Cox [2] has suggested that for a low-power

personal communications network (PCN), quaternary phase shift keying (QPSK) with Nyquist pulse shaping with an excess bandwidth of $\beta = 0.5$ is a good compromise between spectrum efficiency and detectability.

In this paper, we analyze the performance of QPSK in the presence of co-channel interference. Three approaches that can be used for analyzing the co-channel interference are investigated. These are a precise method based on the average probability of error, a sum of sinusoids (sinusoidal) model, and a Gaussian interference model. System performance is assessed in terms of the average bit error rate and the interference penalty. The latter is defined as the increase in signal-to-noise power ratio (SNR) required by a system with co-channel interference in order to maintain the same BER as a system without interference. The examples consider the outdoor microcellular fading environment. In this environment, the fading experienced by the interfering signals may be represented by a Rayleigh-fading model while the fading experienced by the desired signal may be represented by a Ricean- or a Nakagami-fading model.

A Gaussian interference model was used in [3] in a fading environment. This model assumed that all interfering signals had aligned symbol timings and did not consider cross channel ISI effects. Similar assumptions were made in Gaussian interference models used in [5] and [6].

The contributions of this paper are the following. We compute the average BER of QPSK in the presence of co-channel interference on both AWGN and flat fading channels for Nyquist pulse shaping. Using a new method recently published in [15], we do a precise analysis of co-channel interference including the effects of ISI produced by the co-channel interference and the effects of random symbol and carrier timing offsets. The analyses are not limited to a single interferer case, but rather assume the presence of multiple independent co-channel interferers. We also derive new expressions that approximate the BER of these systems by using Gaussian models. These new results differ from past results in that they have a dependence on

the excess bandwidth. The accuracies of our Gaussian models and the sinusoidal models are assessed. Finally, two fading models (Nakagami and Ricean) are used to represent the fading experienced by the desired signal while a Rayleigh-fading model is used to represent the fading experienced by the interfering signals. These models may be used, in particular, to model different outdoor microcellular fading environments.

II. NOISE MODELS

The appearance of the noise and its effect is related to its characteristics. Noise signals can be either periodic in nature or random. Usually observed noise signals during signal transmission are random in nature resulting in abrupt local changes in the transmitting sequence. These noise signals cannot be adequately described in terms of the commonly used Gaussian noise model. The ambient noise is known through experimental measurements to be non-Gaussian due to the impulsive nature of man-made electromagnetic interference, such as car ignition systems and industrial machines in the vicinity of the signal receiver and a great deal of natural noise as well, such as lightning in the atmosphere and ice cracking in the antarctic region which generate non-Gaussian, long-tailed type of noise.

Combined man-made and natural radio noise in analytical model serves following purposes:

- a) It provides a realistic and quantitative description of man-made and natural electromagnetic (EM) Interferences,
- b) It guides experimental protocols for the measurement of such interferences,
- c) It can be used to identify optimal communication systems and their performance comparison with the sub optimal systems.

III. SYSTEM MODELS

A. Transmitted Signal

The QPSK transmitted signal can be written as

$$S(t) = S_a(t) \sin(2\pi f_c t) + S_b(t) \cos(2\pi f_c t) \quad (1)$$

where $S_a(t)$ and $S_b(t)$ are the baseband signals on the in-phase and quadrature paths, respectively, and f_c is the carrier frequency. The signals $S_a(t)$ and $S_b(t)$ are given by

$$\begin{aligned} S_a(t) &= \sum_{k=-\infty}^{\infty} a_k g_T(t - kT), \\ S_b(t) &= \sum_{k=-\infty}^{\infty} b_k g_T(t - kT) \end{aligned} \quad (2)$$

Where $g_T(t)$ is the impulse response of the transmitter filter, and T is the symbol interval. The transmitter filter is assumed to have a square root raised cosine frequency response with a roll-off factor β ($0 \leq \beta \leq 1$). The data bits a_k and b_k ($k = 0, \pm 1, \pm 2, \dots$) can assume values of $\{-1, +1\}$ with equal probabilities. Also, a_k , a_j , b_k and b_j ($-\infty \leq k \leq \infty, -\infty \leq j \leq \infty$) are assumed to be mutually independent.

B. Received Signals

Assume that L co-channel interferers are present. Hence, the total received signal $r(t)$ can be written as

$$r(t) = S_r(t) + \sum_{i=1}^L S_i(t) + n_w(t) \quad (3)$$

Where $S_r(t)$ and $S_i(t)$ represent the contributions from the desired signal and the i th interfering signal, respectively, and $n_w(t)$ is a zero-mean AWGN with two-sided power spectral density of $N_0/2$ W/Hz. Without loss of generality, it has been assumed that zero delay exists between the received signal and the transmitted signal. We consider first a precise interference model. In this model, the interfering signals are assumed to have a similar modulation to that of the desired signal. That is, the interferers are QPSK modulated signals with the same symbol rate, but with random carrier phase shifts and random symbol timing offsets relative to the desired signal. The second approach considered is a sinusoidal interference model. In this model, the resultant interfering signal is represented by a sum of sinusoids with constant (unfaded) amplitudes and unequal random phases. This

model has been used to examine the performances of PSK modulations in the presence of co-channel interference on AWGN channels [7]-[8]. The third model is the Gaussian interference model. In this model, the total interference contribution is represented by a Gaussian noise with mean and variance equal to the mean and variance of the sum of the interfering signals.

The desired signal $S_r(t)$ can be written as

$$\begin{aligned} S_r(t) &= R_s S_a(t) \sin(2\pi f_c t + \theta) \\ &\quad + R_s S_b(t) \cos(2\pi f_c t + \theta) \end{aligned} \quad (4)$$

Where R_s represents the channel gain amplitude affecting the desired signal and the phase θ includes transmitter-to-receiver carrier phase differences and the random phase introduced by the fading channel. Using the precise interference model, $S_i(t)$ can be written as

$$\begin{aligned} S_i(t) &= R_i S_{c_i}(t) \sin(2\pi f_c t + \alpha_i) \\ &\quad + R_i S_{d_i}(t) \cos(2\pi f_c t + \alpha_i) \end{aligned} \quad (5)$$

Where $S_{c_i}(t)$ and $S_{d_i}(t)$ represent the baseband in-phase and quadrature components of the i th interfering signal, respectively.

C. The Detection Process

At the receiver, the total received signal is split into an in-phase component and a quadrature component and detection is then performed. A block diagram of a typical receiver structure can be found in [9]. In the following, we consider only detection in the in-phase path. From symmetry, the results for the quadrature component are similar.

For optimum coherent detection, the received signal is multiplied by locally generated quadrature carriers with a phase that approximates that of the desired received signal. In this paper, perfect carrier phase tracking is assumed for the desired signal. We also assume that frequency tracking and symbol synchronization are perfect for the desired

signal. In practical systems it is difficult to achieve frequency and phase tracking in fading. Furthermore, the interference will affect the synchronization components [10]. Therefore, our results represent a best case situation. Without loss of generality, it is also assumed that $\theta = 0$. The in-phase demodulated signal component is then given by [9]

$$X_a(t) = 2r(t) \sin(2\pi f_c t). \tag{6}$$

Using the precise interference model, it can be shown that above equation can be written as

$$\begin{aligned} X_a(t) = & R_s S_a(t) [1 - \cos(4\pi f_c t)] + R_s S_b(t) \sin(4\pi f_c t) \\ & + \sum_{i=1}^L R_i \{ S_{c_i}(t) [\cos(\alpha_i) - \cos(4\pi f_c t + \alpha_i)] \\ & + S_{d_i}(t) [\sin(4\pi f_c t + \alpha_i) - \sin(\alpha_i)] \} \\ & + 2n_w(t) \sin(2\pi f_c t). \end{aligned} \tag{7}$$

IV. ERROR RATES ON FADING CHANNELS

In this paper, the interfering signals are assumed to experience Rayleigh fading and the desired signal is assumed to experience Ricean or Nakagami fading. These models are valid when the fadings experienced by the distant interfering signals are more severe than that experienced by the desired signal. This may, especially, be true in outdoor microcellular systems where a line of sight component may exist between the transmitter and the intended receiver. In the following, we examine three approaches for computing the average BER on fading channels in the presence of multiple independent co-channel interferers. The first approach is based on using the precise interference model of the previous section. The second and third approaches are based on extending the sinusoidal and the Gaussian interference models to fading conditions.

A. Precise Interference Model

In fading conditions, the amplitudes of the desired signal (R_s) and the interfering signals $\{R_i\} = \{R_1, R_2, \dots, R_L\}$ are random variables. Assuming that $\{R_s, R_1, \dots, R_L\}$ are constants, the conditional BER is given by

(9) where the conditioning is on the desired and interfering signals amplitudes. To include the effects of fading, the conditional BER is averaged over all values of $\{R_s, R_i\} = \{R_s, R_1, \dots, R_L\}$. In this paper, $\{R_s, R_i\}$ are assumed to be independent RV's and $\{R_i\}$ are assumed to be iid RV's. Also, the fading is assumed to be frequency-nonselctive and slow enough such that R_s, R_i , and $\alpha_i (i = 1, 2, \dots, L)$ are constants over the duration of the symbol to be detected and the sequence of symbols interfering with its detection. The fading envelope of the sth interfering signal R_i , is a Rayleigh distributed RV. The probability density function (pdf) of R_i , is given by

$$f_{R_i}(r) = \frac{2r}{\Omega_i} \exp\left(-\frac{r^2}{\Omega_i}\right), \quad \Omega_i = E[R_i^2] \tag{8}$$

The average BER can be written as

$$P_e \cong \frac{1}{2} - \frac{2}{\pi} \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} \frac{e^{-n^2 w^2 / 2}}{n} B_n A_n^L \tag{9}$$

This equation can be used to compute precisely the average BER of QPSK in the presence of L independent Rayleigh-fading QPSK interferers with B_n , for a Nakagami fading desired signal and for a Ricean fading desired signal. A double numerical integration is required to compute A_n .

B. Sinusoidal Interference Model

In this case, the decision RV can be written as

$$y_a = a_o R_s + z + n(0) \tag{10}$$

Therefore, the conditional BER can be written as

$$P_{e|R_s} = Q(R_s / \sqrt{\epsilon}) \tag{11}$$

Where $Q(x)$ is the Q-function. The unconditional BER can be written as

$$P_e = \int_0^\infty P_{e|R_s=r} f_{R_s}(r) dr. \tag{12}$$

Define the average signal-to-noise power ratio (SNR_{av}) and the average signal-to-average total interference power ratio (SIR_{av}) as

$$SNR_{av} = \Omega_s, \quad SIR_{av} = \Omega_s / \sum_{i=1}^L \Omega_i = \Omega_s / (L\Omega_i). \tag{13}$$

Figure shows the average BER of a Nakagami-fading desired signal versus SNR_{av} in the presence of 1 and 6 co-channel interferers with $SIR_{av} = 10$ dB.

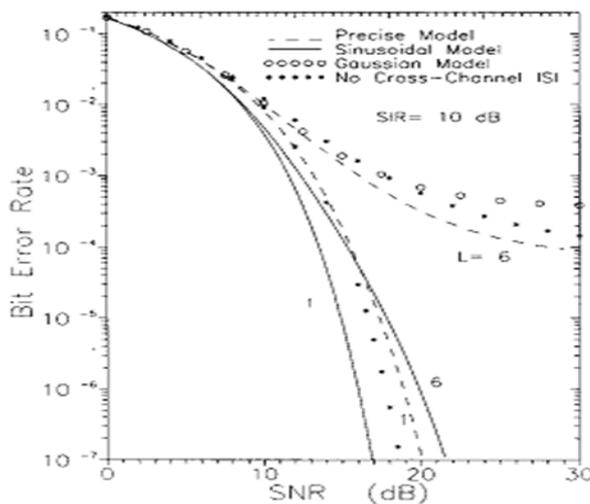


Figure: The average BER of QPSK in AWGN with L co-channel interferers and $SIR = 10$ dB

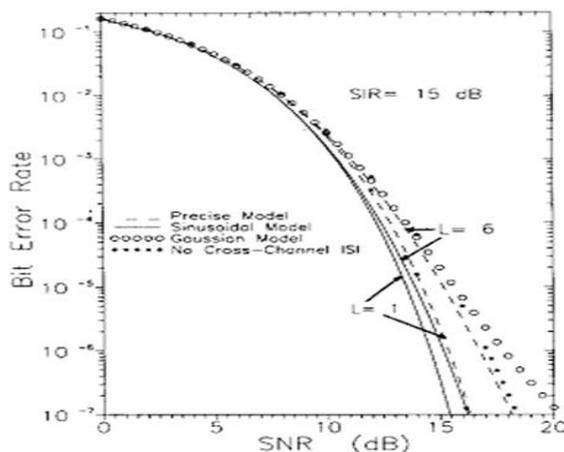


Figure: The average BER of QPSK in AWGN with L co-channel interferers and $SIR = 15$ dB

It is well known that, in general, the Gaussian interference model overestimates the BER in AWGN (without fading) interestingly, the results from the precise model indicate that the reverse trend (the BER is underestimated) is generally true in the fading situations considered in this paper, especially for the case of $L = 1$. This is plausible in light of the following. The Gaussian approximation is exact for unmodulated. Rayleigh-fading interfering signals. The modulation causes, on average, symmetric perturbations in the baseband recovered signal about its unmodulated value. The upward convexity of the Q-function combined with Jensen's inequality leads one to anticipate that the fading BER with modulation is greater than without modulation.

V. CONCLUSION

In this paper, we have analysed the performance of band-width efficient QPSK in the presence of co-channel interference in both nonfading and fading environments. In an AWGN environment, the Gaussian interference model can be an excellent approximation for the case of six interferers. For the case of a single dominant interferer, the Gaussian model is not accurate for all practical values of SNR. The sinusoidal interference model seems to always underestimate the effects of interference. It was seen that for a fixed total interference power, the BER obtained when the total interference power is concentrated in a single interferer is smaller than that obtained when the total interference power is equally distributed among six interferers. The results also showed that for a fixed SIR, the interference penalty increases as the SNR increases. The performance of bandwidth efficient QPSK on fading microcellular channels with co-channel interference was analyzed. A Gaussian model having an explicit dependence on the excess bandwidth was derived. It was seen that for the case of six interferers, the Gaussian model gives an excellent approximation for most practical values of SNR. For the single interferer case, the Gaussian model is good for small to medium values of SNR but is not so accurate for large values of SNR. It was also seen that the sinusoidal interference model can significantly underestimate the effects of co-channel

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